

## HUREWICZ FIBRATIONS

As I mentioned yesterday, it's sometimes useful to think of families of types as akin to Hurewicz fibrations. Here's the definition:

A continuous map  $p : E \rightarrow B$  of topological spaces is a *Hurewicz fibration* if for every space  $X$ , any homotopy  $F : X \times I \rightarrow B$ , and continuous map  $f : X \rightarrow E$ , such that  $p \circ f = F \circ \sigma_0$ , there is a homotopy  $\tilde{F} : X \times I \rightarrow E$  such that  $f = \tilde{F} \circ \sigma_0$  and  $F = p \circ \tilde{F}$ :

$$\begin{array}{ccc} X & \xrightarrow{f} & E \\ \sigma_0 \downarrow & \nearrow \tilde{F} & \downarrow p \\ X \times I & \xrightarrow{F} & B \end{array}$$

Here,  $I = [0, 1]$  is the topological interval, and  $\sigma_0 : X \rightarrow X \times I$  sends  $x$  to  $(x, 0)$ .

This property is called the *homotopy lifting property* and is equivalently expressed as saying that  $p$  has the *right lifting property* with respect to all maps of the form  $\sigma_0$ .

Optional(!) exercise: Show that every covering map is a Hurewicz fibration.

## SOME NOTES ON JUDGMENTS

For more discussions on judgments, derivations, etc., I'll refer you to works by Per Martin-Löf [3–5], Bob Harper [2, Ch. 1–3], and Granström [1] (who was a student of Per Martin-Löf).

In the next 2–4 weeks we're defining a particular formal system in the manner of Per Martin-Löf (many variations are possible). It's a kind of definition that is unfamiliar, so at the same time we're discussing how to *think* about it, as well as how to *prove* something about it (e.g., give example derivations and indicate how to model parts of it).

The formal system is a *model* of informal mathematics: the atomic judgments say that we can construct a certain *type* or a certain *element* of a type (and we can judge some types or elements to be (judgmentally) equal).

## SOME NOTES ON JUDGMENTAL EQUALITY

One way of thinking about judgmental equality vs identifications is via Frege's distinction between *Sinn* and *Bedeutung* (1892). These are translated into English as *sense* and *reference*, or *mode of presentation* and *denotation*. The classical example is

the morning star = the evening star,

where the identification is non-trivial because the two sides have different senses/give different modes of presentation.

In term of groupoids, consider the (set-presented) groupoid  $X$  with objects  $\{1, 2\}$  and identifications  $X(1, 1) = X(1, 2) = X(2, 1) = X(2, 2) = \{\star\}$ . Here, 1 and 2 have different senses, but the same reference. And this is reflected in the fact that  $X$  is equivalent to the trivial groupoid with one object  $T$ . (So indeed  $X$  and  $T$ , as groupoids, themselves have different senses, but the same reference!)

For another example, the propositions “ $2 + 2 = 4$ ” and “for any integer  $n > 2$ , there are no non-trivial integral solutions to  $a^n + b^n = c^n$ ” have very different senses, but the same reference: truth.

## REFERENCES

- [1] Johan Georg Granström. *Treatise on Intuitionistic Type Theory*. Vol. 22. Logic, Epistemology, and the Unity of Science. Springer, Dordrecht, 2011. DOI: 10.1007/978-94-007-1736-7.
- [2] Robert Harper. *Practical Foundations for Programming Languages*. 2nd ed. Cambridge University Press, 2016, p. 512. URL: <http://www.cs.cmu.edu/~rwh/pfpl/>.
- [3] Per Martin-Löf. *Intuitionistic type theory. Notes by Giovanni Sambin*. Vol. 1. Studies in Proof Theory. Bibliopolis, 1984, pp. iv+91. URL: <http://archive-pml.github.io/martin-lof/pdfs/Bibliopolis-Book-retypeset-1984.pdf>.
- [4] Per Martin-Löf. “On the Meanings of the Logical Constants and the Justification of Logical Laws”. In: *Nordic Journal of Philosophical Logic* 1 (1). Notes for three lectures given in Siena, April 1983, pp. 11–60. URL: <http://archive-pml.github.io/martin-lof/pdfs/Meanings-of-the-Logical-Constants-1983.pdf>.
- [5] Per Martin-Löf. “Truth of a Proposition, Evidence of a Judgement, Validity of a Proof”. In: *Synthese* 73 (1), pp. 407–420. URL: <http://archive-pml.github.io/martin-lof/pdfs/Truth-of-a-Proposition-Evidence-of-a-Judgment-1987.pdf>.