This week, we'll finish the various characterizations of equivalence (§ 8), prove the "fundamental theorem of identity types" (§ 9), and start work on the truncation level hierarchy (§ 10).

We skip Theorem 9.1.6, and part (iii) of Theorem 9.2.2 (on "identity systems").

Equivalences

Fix $f : A \to B$. The following types are all logically equivalent ($\leftrightarrow$):

- $\text{has-inverse}(f) :\equiv \sum_{(g:B\to A)}(f \circ g \sim \text{id}_B) \times (g \circ f \sim \text{id}_A)$,
- $\text{is-equiv}(f) :\equiv \text{sec}(f) \times \text{ret}(f)$,
- $\text{is-contr-map}(f) :\equiv \prod_{(y:B)} \text{is-contr}(f^{-1}\, y)$,
- $\text{is-coh-invertible}(f) :\equiv \sum_{(g:B\to A)} \sum_{(G:f\circ g\sim \text{id}_B)} \sum_{(H:g\circ f\sim \text{id}_A)} (G \cdot f \sim f \cdot H)$,
- $\text{path-split}(f) :\equiv \text{sec}(f) \times \prod_{(x,y:A)} \text{sec}(\text{ap}_f(x,y))$ (cf. Ex. 9.9)

We'll later see that all but the first are propositions. It's of course useful to have many different characterizations of an important concept.

Exercises

As always, the numbers refer to Rijke's *Introduction to Homotopy Type Theory*.

- Ex. 8.6: The fibers of a family are (homotopy) fibers.
- Ex. 8.7: We can replace a map by the family of its fibers.
- Ex. 9.6–9.9: Embeddings and the path-split version of equivalence.